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Ex: 1.1

Q. 1 Determine whether each of the following relations are reflexive, symmetric and transitive.

(iii) Relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as

$R = \{(x, y) : y \text{ is divisible by } x\}$.

Soln: (a) For reflexive,

We know that y is divisible by x,

$\forall x \in A$.

$\therefore (x, x) \in R, \forall x \in R$. So, R is reflexive.

(b) For symmetry,

We observe that 6 is divisible by 2.

This means that $(2, 6) \in R$ but $(6, 2) \notin R$. So, R is not symmetric.

(c) For transitivity,

Let $(x, y) \in R$ and $(y, z) \in R$, then z is divisible by x

$\Rightarrow (x, z) \in R$

For example, 2 is divisible by 1, 4 is divisible by 2

So, 4 is divisible by 1. So, R is transitive.

(iv) Relation R in the set Z of all integers defined as $R = \{(x, y) : xy \text{ is an integer}\}$

Soln: (a) For reflexive,

Put $y = x$, $x \cdot x = 0$, which is an integer $\forall x \in Z$.

So, R is reflexive on Z.

(b) For symmetry,

Let $(x,y) \in R$, then $(x-y)$ is an integer say λ and also $y-x = -\lambda$ [$\because \lambda \in Z \Rightarrow -\lambda \in Z$]

$\therefore y-x$ is an integer $\Rightarrow (y,x) \in R$. So, R is symmetric.

(c) For transitive,

Let $(x,y) \in R$ and $(y,z) \in R \Rightarrow (x,z) \in R$.

That is, $x-y$ = integer and $y-z$ = integer then $x-z$ is an integer. So, R is transitive.

(V) Relation R in set of human beings in a town at a particular time given by

Q. (a) $R = \{(x,y) : x \text{ and } y \text{ work to the same place}\}$

Soln: It is clear that R is reflexive, symmetric and transitive obviously.

Q. (b) $R = \{(x,y) : x \text{ and } y \text{ live in the same locality}\}$

Soln: It is clear that R is reflexive, symmetric and transitive obviously.

Q. (c) $R = \{(x,y) : x \text{ is exactly 7 cm taller than } y\}$

Soln: It is clear that R is not reflexive as x is not 7 cm taller than x.

R is not symmetric as if x is exactly 7 cm taller than y, then y cannot be 7 cm taller than x and R is not transitive as if x is exactly 7 cm taller than y and y is exactly 7 cm taller than z, then x is exactly 14 cm taller than z.

(d) It is clear that R is not reflexive as x is not wife of x. R is not symmetric, as if x is wife of y, then y is husband of x and R is transitive as transitivity is not contradicted in this case. Whenever, $(x,y) \in R$, then $(y,z) \notin R$ for any z as if x is wife of y, then y is a male and a male cannot be a wife.